EXERCISE-1

SECTION-A [FUNDAMENTAL]

Q.1 Simplify :

(i) $\log\left(\frac{1}{x}+\frac{1}{y}\right)-\log(x+y)+\log x+\log y.$ (ii) $\frac{\log x^3z-\log zy^3}{\log x-\log y}$.

Q.2 Show that :

(i) $\log 2+16\log\frac{16}{15}+12\log\frac{25}{24}+7\log\frac{81}{80}=1$

(ii) $\log\frac{81}{8}-2\log\frac{3}{2}+3\log\frac{2}{3}+\log\frac{3}{4}=0$

Q.3 Solve for x :

(i) $\log(x+1)+\log(x-1)=\log 99$ (ii) $\frac{\log 16}{\log 4}=\log x$

(iii) $\log(3x-2)+\log(3x+2)=5\log 2$ (iv) $\log 5+\log(5x+1)=\log(x+5)+1$

Q.4 Simplify each of the following :

(i) $\log_8\sqrt{6}+\log_8\left(\sqrt{\frac{2}{3}}\right)-\log_8(\log_3 9)$ (ii) $\log_2[\log_2\{\log_3(\log_3 27^3)\}]$

Q.5 (i) If $\log\left(\frac{a-b}{2}\right)=\frac{1}{2}(\log a+\log b)$, show that $a^2+b^2=6ab$

(ii) If $\log\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log a+\log b)$, show that $\frac{a+b}{2}=\sqrt{ab}$ and $a^2+b^2=2ab$.

(iii) If $a^2+b^2=7ab$, prove that $\log\left(\frac{a+b}{3}\right)=\frac{1}{2}(\log a+\log b)$.

Q.6 (i) If $a=\log_{24}12$, $b=\log_{36}24$ and $c=\log_{48}36$, show that $1+abc=2bc$.

(ii) If $x=\log\frac{2}{3}$, $y=\log\frac{3}{5}$ and $z=\log\frac{5}{2}$, show that $x+y+z=0$

(iii) If $y=x^{\frac{1}{m}}$, show that $m=\log_y x$.

Q.7 Prove that

(i) $\log_3\log_2\log_{\sqrt{3}}81=1$

(ii) $\log_a x\times\log_b y=\log_b x\times\log_a y$

(iii) $\log_2\log_2\log_2 16=1$

(iv) $\log_a x=\log_b x\times\log_c b\times\ldots\ldots\times\log_n m\times\log_a n$

Q.8 Simplify:

$$\frac{\log_9 11}{\log_5 13} \div \frac{\log_3 11}{\log_{\sqrt{5}} 13}$$

Q.9 (i) If $\log_4 10 = x$, $\log_2 20 = y$ and $\log_5 8 = z$. Prove that $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$.

(ii) If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$. Prove that $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$.

Q.10 (i) Prove that $\frac{1}{1+\log_b a + \log_b c} + \frac{1}{1+\log_c a + \log_c b} + \frac{1}{1+\log_a b + \log_a c} = 1$.

(ii) Show that $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots\ldots + \frac{1}{\log_{43} n} = \frac{1}{\log_{43!} n}$.

SECTION-B [JEE (MAIN)]

Q.1 The value of $\log_{10}2 + 16\log_{10}\left(\frac{16}{15}\right) + 12\log_{10}\left(\frac{25}{24}\right) + 7\log_{10}\left(\frac{81}{80}\right)$ is equal to

(A) $\log_{10}2$ (B) $\log_{10}5$ (C) 0 (D) 1

Q.2 The value of $2^{\log_3 5} - 5^{\log_3 2} + 3^{\log_2 7} - 7^{\log_2 3} + 6^{\log_6 11} - 4^{\log_2 3}$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

Q.3 If $m = \log_3 2$, then $\log_{27}24$ equals

(A) 3m (B) 3m + 1 (C) $\frac{m+1}{3}$ (D) $\frac{3m+1}{3}$

Q.4 $\sum_{n=1}^{1023}\log_2\left(1+\frac{1}{n}\right)$ equals

(A) 8 (B) 10 (C) 1024 (D) 1023

Q.5 If a, b and c (where c is the largest of the three numbers) are the sides of a right angled triangle, then

(A) $\log_{c^2}(a^2+b^2) = 0$ (B) $\log_{a^2}(c^2-b^2) = 0$

(C) $\log_c(a^2+b^2) = 2$ (D) $\log_a(b^2-c^2) = 2$

Q.6 If $2\log_2\log_2 x + \log_{1/2}\log_2(2\sqrt{2}\,x) = 1$, then the number of possible real values of x is

(A) 1 (B) 2 (C) 3 (D) 0

Q.7 The value of the expression $6^{\frac{\log_5(\log_3 80)}{\log_5 6}}$ always lies between

(A) 1 and 2 (B) 3 and 4 (C) 5 and 6 (D) 7 and 8

Q.8 The number of positive integers having characteristic 4, if the base of logarithm is 3, is equal to

(A) 161 (B) 162 (C) 163 (D) 164

Q.9 If S = $\log_a\left(\frac{1}{2}\right)+\log_a\left(\frac{2}{3}\right)+ \ ....+\log_a\left(\frac{1330}{1331}\right)$ and a = 11 then S equals

(A) 1 (B) –2 (C) 3 (D) –3

Q.10 The number $N=\frac{\log_5 250}{\log_{50} 5}-\frac{\log_5 10}{\log_{1250} 5}$, when simplified reduces to a natural number, then N equals

(A) 2 (B) 3 (C) 4 (D) 8

Q.11 The value of $\log_7(343) - \log_7(1001) + \log_7(143)$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

Q.12 Let T = $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$ then $\log_{625}(T)$ equals

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) None of these

Q.13 If x = $\log_5\left(\log_5\left(\sqrt{5\sqrt{5\sqrt{5}}}\right)\right)$ then x is equal to

(A) 0 (B) 1 (C) $\log_5(7/8)$ (D) $\log_5(8/7)$

Q.14 The value of $\log_{\sqrt{2}}\left(\text{anti}\log_{128}\left(\frac{3}{7}\right)\right)$ equals.

(A) 16 (B) 8 (C) 6 (D) 4

Q.15 The value of $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) +...+ \log_{10}(\log_{1023} 1024)$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

Q.16 The value of $\log_2\left((\log_{81} 3)^{\log_3 81}\right)$ is equal to

(A) – 8 (B) $-4\log_2 3$ (C) 8 (D) $-4\log_3 2$

Q.17 If $N_1 = 5^7 + 5^7 + 5^7 + 5^7 + 5^7$
and $N_2 = 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3 + 10^3$
then the value of $\log_{250} N_1 N_2$ is equal to

(A) 4 (B) 8 (C) 10 (D) 21

Q.18 $x = \sqrt{3\sqrt{3\sqrt{3.....\infty}}}$ and $y = \sqrt[3]{4\sqrt[3]{4\sqrt[3]{4.....\infty}}}$ then which of the following holds good?

(A) $x = y$ (B) $2x = 3y$ (C) $3x = 2y$ (D) $x < y$

Q.19 If $3^{\log_{\sqrt{3}}\sqrt{x}} + 3^{\log_3 x} + 27^{\log_{3^2} x^2} = 3$ then the value of x is equal to

(A) 1 (B) 3 (C) 9 (D) 27

Q.20 The value of $\log_{\sqrt{b}} a \log_{c^{1/3}} b \log_{a^{1/4}} c$ is equal to

(A) 12 (B) 24 (C) $\frac{1}{12}$ (D) $\frac{1}{24}$

Q.21 The value of $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1}$ is equal to

(A) –3 (B) –1 (C) 1 (D) 3

Q.22 The value of $2016^{\log_{2015} 2017} - 2017^{\log_{2015} 2016}$ is equal to

(A) 0 (B) – 1 (C) 1 (D) – 2

Q.23 If $a^{13} \cdot b^{15} = 1$ and $\log_a(a^{15}b^{13}) = \frac{p}{q}$, where p & q are co-prime then (p + q) equals

(A) 16 (B) 28 (C) 70 (D) 71

Q.24 The number of value of x satisfying $|x - 2| + \sqrt{x - 4} = 0$ is/are

(A) 1 (B) 0 (C) 2 (D) More than 2

Q.25 The number of zeros after decimal and before first significant digit in 50^{-100} is equal to, $(\log_{10}(2) = 0.3010)$

(A) 168 (B) 169 (C) 170 (D) 171

EXERCISE-2

[JEE ADVANCED]

[SINGLE CORRECT CHOICE TYPE]

Q.1 For $N > 1$, the product $\frac{1}{\log_2 N} \cdot \frac{1}{\log_N 8} \cdot \frac{1}{\log_{32} N} \cdot \frac{1}{\log_N 128}$ simplifies to

(A) $\frac{3}{7}$ (B) $\frac{3}{7 \ln 2}$ (C) $\frac{3}{5 \ln 2}$ (D) $\frac{5}{21}$ **[3010110244]**

Q.2 If p is the smallest value of x satisfying the equation $2^x + \frac{15}{2^x} = 8$ then the value of 4^p is equal to

(A) 9 (B) 16 (C) 25 (D) 1 **[3010110950]**

Q.3 The sum of two numbers a and b is $\sqrt{18}$ and their difference is $\sqrt{14}$. The value of $\log_b a$ is equal to

(A) – 1 (B) 2 (C) 1 (D) $\frac{1}{2}$ **[3010112439]**

Q.4 The value of the expression $(\log_{10}2)^3 + \log_{10}8 \cdot \log_{10}5 + (\log_{10}5)^3$ is

(A) rational which is less than 1 (B) rational which is greater than 1

(C) equal to 1 (D) an irrational number **[3010111646]**

Q.5 Let $N = 10^{3\log 2 - 2\log\left(\log 10^3\right) + \log\left(\left(\log 10^6\right)^2\right)}$ where base of the logarithm is 10. The characteristic of the logarithm of N to the base 3, is equal to

(A) 2 (B) 3 (C) 4 (D) 5

[3010112388]

Q.6 If $x = \frac{\sqrt{10}+\sqrt{2}}{2}$ and $y = \frac{\sqrt{10}-\sqrt{2}}{2}$, then the value of $\log_2(x^2 + xy + y^2)$, is equal to

(A) 0 (B) 2 (C) 3 (D) 4 **[3010112337]**

Q.7 Suppose that $x < 0$. Which of the following is equal to $\left|2x \quad \sqrt{(x \quad 2)^2}\right|$

(A) x – 2 (B) 3x – 2 (C) 3x + 2 (D) – 3x + 2

[3010112438]

Q.8 If $N = \left(2^{\log_{70} 9800}\right)\left(5^{\log_{70} 140}\right)\left(7^{\log_{70} 2}\right)$, then N is equal to

(A) 20 (B) 60 (C) 18 (D) 40

[3010111001]

Q.9 The expression $q^{\left(\sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}\right)^{\log_3 p^{\left[\frac{\log_q(\log_q r)}{(\log_q p)}\right]}}}$ simplified to

(A) p (B) q (C) r (D) 3

[3010111242]

Q.10 If a, b are co-prime numbers and satisfying $\left(2+\sqrt{3}\right)^{\frac{1}{\log_a\left(2-\sqrt{3}\right)}+\frac{1}{\log_b\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}}=\frac{1}{12}$,

then (a + b) can be is equal to

(A) 13 (B) 5 (C) 7 (D) 8

[3010110590]

Q.11 If $2^{(\log_2 3)^x}=3^{(\log_3 2)^x}$ then the value of x is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$ **[3010110840]**

Q.12 The least value of the expression $2\log_{10}x-\log_x(0.01)$, for $x>1$ is :

(A) 10 (B)2 (C) –0.01 (D) None of these

[3010110151]

Q.13 The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}}=\sqrt{2}$ has :

(A) at least one real solution (B) exactly three real solution

(C) exactly one irrational (D) Complex roots **[3010110651]**

Q.14 If $\alpha^{\log_2 3}=4$, $\beta^{\log_3 5}=3$ and $\gamma^{\log_5 7}=25$ then the value of $\alpha^{(\log_2 3)^2}+\beta^{(\log_3 5)^2}+\gamma^{(\log_5 7)^2}$ is equal to

(A) 58 (B) 63 (C) 83 (D) 39

[PARAGRAPH TYPE]

Paragraph for Question no. 15 to 17

A denotes the product xyz where x, y and z satisfy

$\log_3 x=\log 5-\log 7$

$\log_5 y=\log 7-\log 3$

$\log_7 z=\log 3-\log 5$

B denotes the sum of square of solution of the equation

$\log_2(\log_2 x^6-3)-\log_2(\log_2 x^4-5)=\log_2 3$

C denotes characteristic of logarithm

$\log_2(\log_2 3)-\log_2(\log_4 3)+\log_2(\log_4 5)-\log_2(\log_6 5)+\log_2(\log_6 7)-\log_2(\log_8 7)$

Q.15 The value of A + B + C, is

(A) 18 (B) 34 (C) 32 (D) 24

Q.16 The value of $\log_2 A + \log_2 B + \log_2 C$, is

(A) 5 (B) 6 (C) 7 (D) 4

Q.17 The value of $|A - B + C|$

(A) – 30 (B) 32 (C) 28 (D) 30

[3010112328]

[MULTIPLE CORRECT CHOICE TYPE]

Q.18 Let $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is

(A) a natural number (B) coprime with 3

(C) a rational number (D) a composite number **[3010112387]**

Q.19 If $\left(a^{\log_b x}\right)^2 - 5\, x^{\log_b a} + 6 = 0$, where $a > 0$, $b > 0$ & $ab \neq 1$, then the value of x can be equal to

(A) $2^{\log_b a}$ (B) $3^{\log_a b}$ (C) $b^{\log_a 2}$ (D) $a^{\log_b 3}$ **[3010112336]**

Q.20 Which of the following statement(s) is/are true ?

(A) $\log_{10} 2$ lies between $\frac{1}{4}$ and $\frac{1}{3}$

(B) $\log_{\text{cosec}\left(\frac{\pi}{6}\right)}\left(\cos\frac{\pi}{3}\right) = -1$

(C) $e^{ln\,(ln3)}$ is smaller than 1

(D) $\log_{10} 1 + \frac{1}{2}\log_{10} 3 + \log_{10}\left(2+\sqrt{3}\right) = \log_{10}\left(1+\sqrt{3}+\left(2+\sqrt{3}\right)\right)$ **[3010112432]**

Q.21 The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has

(A) one irrational solution (B) no prime solution

(C) two real solutions (D) no integral solution **[3010112381]**

Q.22 Which of the following real numbers is(are) non-positive ?

(A) $\log_{\frac{1}{2}}\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)$ (B) $\log_{12}\left(\sqrt{65}-7\right)$

(C) $\log_2 \log_5 3 \cdot \log_7 5 \cdot \log_3 7$ (D) $\log_7\left(\frac{3}{2}\right)^{\frac{-2}{3}}$ **[3010112330]**

[MATRIX TYPE]

Q.23

	Column-I	Column-II
(A)	The expression $x = \log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$ simplifies to	(P) an integer
(B)	The number $N = 2^{\left(\log_2 3 \cdot \log_3 4 \cdot \log_4 5 ..... \cdot \log_{99} 100\right)}$ simplifies to	(Q) a prime
(C)	The expression $\frac{1}{\log_5 3}+\frac{1}{\log_6 3}-\frac{1}{\log_{10} 3}$ simplifies to	(R) a natural
(D)	The number $N = \sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$ simplifies to	(S) a composite

[3010112430]

Q.24

	Column-I	Column-II
(A)	The value(s) of x, which does not satisfy the equation $\log_{2^2}(x^2-x)-4\log_2(x-1)\log_2 x = 1$, is (are)	(P) 2
(B)	The value of x satisfying the equation $2^{\log_2 e^{\ln 5^{\log_5 7^{\log_7 10^{\log_{10}(8x-3)}}}}} = 13$, is	(Q) 3
		(R) 4
(C)	The number $N = \left(\frac{1}{\log_2 \pi}+\frac{1}{\log_6 \pi}\right)$ is less than	(S) 5
(D)	Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1+9^{\log_3 8}\right)^{\log_{65} 5}$ then $(l + m)$ is divisible by	(T) 6

[3010112379]

[INTEGER TYPE]

Q.25 Let **A** denotes the value of $\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right) + \log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$ when a = 43 and b = 57

and **B** denotes the value of the expression $\left(2^{\log_6 18}\right)\cdot\left(3^{\log_6 3}\right)$. **[3010111267]**

Q.26 If $k^{\log_2 5} = 16$, find the value of $k^{(\log_2 5)^2}$. **[3010110921]**

Q.27 If mantissa of a number N to the base 32 is varying from 0.2 to 0.8 both inclusive, and whose characteristic is 1, then find the number of integral values of N. **[3010110177]**

Q.28 Let $\log_2 x + \log_4 y + \log_4 z = 2$

$\log_9 x + \log_3 y + \log_9 z = 2$

and $\log_{16} x + \log_{16} y + \log_4 z = 2$.

Find the value of $\frac{yz}{x}$. **[3010110900]**

Q.29 Find the value of x satisfying $\log_{10}(2^x + x - 41) = x(1 - \log_{10} 5)$. **[3010110220]**

Q.30 Positive numbers x, y and z satisfy $xyz = 10^{81}$ and $(\log_{10} x)(\log_{10} yz) + (\log_{10} y)(\log_{10} z) = 468$.
Find the value of $(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2$ **[3010111000]**

Q.31 If $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$. Find the number of integers in :
(a) 5^{200} (b) 6^{15} & (c) the number of zeros after the decimal in 3^{-100}.
[3010111943]

[SUBJECTIVE TYPE]

Q.32 If $x = \log_3 4$ and $y = \log_5 3$, find the value of $\log_3 10$ and $\log_3(1.2)$ in terms of x and y.

Q.33 If 'x' and 'y' are real numbers such that, $2\log(2y - 3x) = \log x + \log y$, find $\frac{x}{y}$.
[3010110291]

Q.34 (a) Given : $\log_{10} 34.56 = 1.5386$, find $\log_{10} 3.456$; $\log_{10} 0.3456$ & $\log_{10} 0.003456$.
(b) Find the number of positive integers which have the characteristic 3, when the base of the logarithm is 7.
(c) If $\log_{10} 2 = 0.3010$ & $\log_{10} 3 = 0.4771$, find the value of $\log_{10}(2.25)$.
(d) If $N = \text{antilog}_3\left(\log_6\left(\text{antilog}_{\sqrt{5}}(\log_5 1296)\right)\right)$, then find the characteristic of $\log N$ to the base 2.
[3010111301]

Q.35 Solve for x the following equation : $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$
[3010110279]

EXERCISE-3

[JEE(ADVANCED) PREVIOUS YEAR]

Q.1 Number of solutions of $\log_4(x-1) = \log_2(x-3)$ is :

(A) 3 (B) 1 (C) 2 (D) 0 **[3010110575]**

[JEE 2001]

Q.2 Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}.$$

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6 **[3010111020]**

[JEE 2011, 3]

Q.3 The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}}.....}}\right)$ is [JEE 2012, 4]

[3010112474]

Q.4 If $3^x = 4^{x-1}$, then x =

(A) $\frac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\frac{2}{2 - \log_2 3}$ (C) $\frac{1}{1 - \log_4 3}$ (D) $\frac{2\log_2 3}{2\log_2 3 - 1}$

[JEE ADV. 2013, 3 (–1)]

EXERCISE-4

[RANK BOOSTER]

[PARAGRAPH TYPE]

Paragraph for question nos. 1 to 3

Let $x = \alpha$ be the value of x satisfying the equation $\log_p(x+32) = 6$, where $p = \sqrt{\sqrt{9+4\sqrt{5}} - \sqrt{9-4\sqrt{5}}}$.

Q.1 The value of $(\log_2\alpha)^2 + 3\log_2\left(\sqrt[5]{\alpha}\right) - \text{antilog}_8\left(\frac{1}{3}\right)$ is equal to

(A) 25 (B) 26 (C) 27 (D) 28

Q.2 Which of the following is **incorrect**?

(A) The characteristic of logarithm of number α^2 with respect to base 2 is 10.

(B) $2^{\log_7(\alpha)} = \alpha^{\log_7(2)}$ (C) $3^{\sqrt{\log_3\alpha}} = \alpha^{\sqrt{\log_\alpha 3}}$

(D) $\log_6(\alpha+4) > \log_5(\alpha-7)$

Q.3 Number of zeroes after decimal in $N = \frac{1}{\alpha^\alpha}$ is equal to

[**Use :** $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$]

(A) 46 (B) 47 (C) 48 (D) 49

[MULTIPLE CORRECT CHOICE TYPE]

Q.4 Identify which of the following statement(s) is(are) correct?

(A) $\log_{\sqrt{2}}(49)\cdot\log_{\sqrt{3}}(2)\cdot\log_{\sqrt{7}}(9) = 2^5$

(B) $2 < \log_\pi(4) + \log_2(2\sqrt{\pi}) < 3$

(C) If $\log^2 x + \log^2 y = (\log(xy))^2$ then number of ordered pairs (x, y) is 2.

(D) If $\log^2 x + \log^2 y = (\log(xy))^2$ then number of ordered pairs (x, y) is infinite.

Q.5 Let N be the number of significant digits in $100e^{23}$ and M be the number of zeros after decimal but before significant digit in e^{-23}. Then which of the following is(are) **correct**?

[**Use:** $\log_{10} e = 0.434$]

(A) N + M = 21 (B) N + M = 23 (C) N – M = 2 (D) N – M = 3

[MATRIX TYPE]

Q.6

	Column-I	Column-II
(A)	The value of $2^{\log_3 5} - 5^{\log_3 2} + 3^{\log_2 7} - 7^{\log_2 3} + 6^{\log_6 11} - 4^{\log_2 3}$ is equal to	(P) 0
(B)	The value of $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$ is equal to	(Q) 1
(C)	The number $N = \frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$, when simplified reduces to a natural number, then N equals	(R) 2
(D)	The value of $\log_{10}\left(\frac{1}{1+x^{m-n}+x^{p-n}} + \frac{1}{1+x^{n-p}+x^{m-p}} + \frac{1}{1+x^{p-m}+x^{n-m}}\right)$ is equal to	(S) 3 (T) 4

[INTEGER TYPE]

Q.7 For x, y ∈ N, if $3^{2x-y+1} = 3^{y-2x+1} - 8$ and $\log_6\left|2x^2y - xy^2\right| = 1 + \log_{36}(xy)$, then find the absolute value of $(x-y)$. **[3010110550]**

Q.8 Find the number of integral solution of the equation $\log_{\sqrt{x}}(x + |x-2|) = \log_x(5x - 6 + 5|x-2|)$. **[3010110092]**

Q.9 Suppose p, q, r and s ∈ N satisfying the relation $p + \dfrac{1}{q + \dfrac{1}{r + \dfrac{1}{s}}} = \dfrac{89}{68}$, then find the value of $(pq + rs)$. **[3010110887]**

Q.10 If $\log_3 M = a_1 + b_1$ and $\log_5 M = a_2 + b_2$ where $a_1, a_2 \in N$ and $b_1, b_2 \in [0, 1)$. If $a_1 a_2 = 6$, then find the number of integral values of M. **[3010111644]**

Q.11 If (x_1, y_1) and (x_2, y_2) are the solution of the system of equation

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1$,

then find the value of $\log_{30}(x_1 y_1 x_2 y_2)$. **[3010112641]**

Q.12 Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$. Find $y \in N$. **[3010112001]**

[SUBJECTIVE TYPE]

Q.13 Prove that : $2^{\left(\sqrt{\log_a \sqrt[4]{ab} + \log_b \sqrt[4]{ab}} - \sqrt{\log_a \sqrt[4]{\frac{b}{a}} + \log_b \sqrt[4]{\frac{a}{b}}}\right) \cdot \sqrt{\log_a b}} = \begin{cases} 2 & \text{if } b \geq a > 1 \\ 2^{\log_a b} & \text{if } 1 < b < a \end{cases}$ **[3010110800]**

Q.14 Find the value of x satisfying the equation,

$$\sqrt{\left(\log_3 \sqrt[3]{3x} + \log_x \sqrt[3]{3x}\right) \cdot \log_3 x^3} + \sqrt{\left(\log_3 \sqrt[3]{x/3} + \log_x \sqrt[3]{3/x}\right) \cdot \log_3 x^3} = 2.$$

[3010112344]

Q.15 Given that $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8) = \dfrac{2}{s^3 + 1}$. Write $\log_2 \dfrac{a^2 b^5}{c^4}$ as a function of 's' (a, b, c > 0, c ≠ 1). **[3010111701]**

ANSWER KEY

EXERCISE-1

SECTION-A

Q.1 (i) 0; (ii) 3 Q.3 (i) x = 10; (ii) x = 100 ; (iii) x = 2; (iv) x = 3

Q.4 (i) 0 ; (ii) 0 Q.8 1

SECTION-B

Q.1	D	Q.2	C	Q.3	D	Q.4	B	Q.5	C
Q.6	A	Q.7	B	Q.8	B	Q.9	D	Q.10	A
Q.11	C	Q.12	B	Q.13	C	Q.14	C	Q.15	B
Q.16	A	Q.17	A	Q.18	B	Q.19	A	Q.20	B
Q.21	C	Q.22	A	Q.23	D	Q.24	B	Q.25	B

EXERCISE-2

Q.1	D	Q.2	A	Q.3	A	Q.4	C	Q.5	B
Q.6	C	Q.7	D	Q.8	D	Q.9	C	Q.10	C
Q.11	A	Q.12	D	Q.13	B	Q.14	B	Q.15	B
Q.16	A	Q.17	D	Q.18	AC	Q.19	BC	Q.20	ABD
Q.21	ABC	Q.22	ACD	Q.23	(A) P ;(B) PRS ; (C) PR ; (D) PQR				
Q.24	(A) QRST ; (B) P ; (C) QRST ; (D) PRS					Q.25	12		
Q.26	625	Q.27	449	Q.28	54	Q.29	41	Q.30	5625

Q.31 (a) 140 ; (b) 12 ; (c) 47 Q.32 $\frac{xy+2}{2y}, \frac{xy+2y-2}{2y}$ Q.33 4/9

Q.34 (a) 0.5386; $\overline{1}.5386$; $\overline{3}.5386$ (b) 2058 (c) 0.3522 (d) 3

Q.35 x = –1/4 is the only solution

EXERCISE-3

Q.1	B	Q.2	C	Q.3	4	Q.4	ABC

EXERCISE-4

Q.1	B	Q.2	D	Q.3	C	Q.4	AD	Q.5	AD
Q.6	(A) R ; (B) P ; (C) R; (D) P			Q.7	5	Q.8	1	Q.9	23
Q.10	54	Q.11	12	Q.12	6	Q.14	$x \in [1/3, 3] - \{1\}$		

Q.15 $2s + 10s^2 - 3(s^3 + 1)$